Further Pure Mathematics FP2

Advanced Subsidiary

Thursday 24 June 2010 – Morning

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Further Pure Mathematics FP2), the paper reference (6668), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

1. (*a*) Express $\dfrac{3}{(3r-1)(3r+2)}$ in partial fractions.

(2)

(*b*) Using your answer to part (a) and the method of differences, show that

$$\sum_{r=1}^{n} \frac{3}{(3r-1)(3r+2)} = \frac{3n}{2(3n+2)}.$$

(3)

(*c*) Evaluate $\displaystyle\sum_{r=100}^{1000} \frac{3}{(3r-1)(3r+2)}$, giving your answer to 3 significant figures.

(2)

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2. The displacement x metres of a particle at time t seconds is given by the differential equation

$$\frac{d^2 x}{dt^2} + x + \cos x = 0.$$

When $t = 0$, $x = 0$ and $\dfrac{dx}{dt} = \dfrac{1}{2}$.

Find a Taylor series solution for x in ascending powers of t, up to and including the term in t^3.

(5)

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3. (*a*) Find the set of values of x for which

$$x + 4 > \frac{2}{x+3}.$$

(6)

(*b*) Deduce, or otherwise find, the values of x for which

$$x + 4 > \frac{2}{|x+3|}.$$

(1)

4.
$$z = -8 + (8\sqrt{3})i$$

(*a*) Find the modulus of z and the argument of z.

(3)

Using de Moivre's theorem,

(*b*) find z^3,

(2)

(*c*) find the values of w such that $w^4 = z$, giving your answers in the form $a + ib$, where $a, b \in \mathbb{R}$.
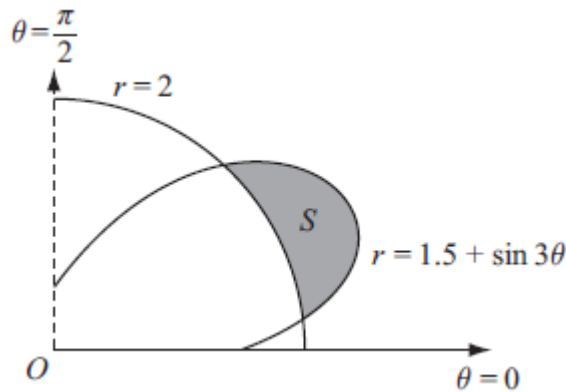
(5)

5.

Figure 1

Figure 1 shows the curves given by the polar equations

$$r = 2, \qquad\qquad 0 \le \theta \le \frac{\pi}{2},$$

$$\text{and} \quad r = 1.5 + \sin 3\theta, \qquad 0 \le \theta \le \frac{\pi}{2}.$$

(*a*) Find the coordinates of the points where the curves intersect.

(3)

The region S, between the curves, for which $r > 2$ and for which $r < (1.5 + \sin 3\theta)$, is shown shaded in Figure 1.

(*b*) Find, by integration, the area of the shaded region S, giving your answer in the form $a\pi + b\sqrt{3}$, where a and b are simplified fractions.

(7)

6. A complex number z is represented by the point P in the Argand diagram.

(a) Given that $|z - 6| = |z|$, sketch the locus of P.

(2)

(b) Find the complex numbers z which satisfy both $|z - 6| = |z|$ and $|z - 3 - 4i| = 5$.

(3)

The transformation T from the z-plane to the w-plane is given by $w = \dfrac{30}{z}$.

(c) Show that T maps $|z - 6| = |z|$ onto a circle in the w-plane and give the cartesian equation of this circle.

(5)

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7. (a) Show that the transformation $z = y^{\frac{1}{2}}$ transforms the differential equation

$$\frac{dy}{dx} - 4y \tan x = 2y^{\frac{1}{2}} \qquad \text{(I)}$$

into the differential equation

$$\frac{dz}{dx} - 2z \tan x = 1 \qquad \text{(II)}$$

(5)

(b) Solve the differential equation (II) to find z as a function of x.

(6)

(c) Hence obtain the general solution of the differential equation (I).

(1)

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8. (*a*) Find the value of λ for which $y = \lambda x \sin 5x$ is a particular integral of the differential equation

$$\frac{d^2 y}{dx^2} + 25y = 3 \cos 5x.$$

(4)

(*b*) Using your answer to part (a), find the general solution of the differential equation

$$\frac{d^2 y}{dx^2} + 25y = 3 \cos 5x.$$

(3)

Given that at $x = 0$, $y = 0$ and $\dfrac{dy}{dx} = 5$,

(*c*) find the particular solution of this differential equation, giving your solution in the form $y = \mathrm{f}(x)$.

(5)

(*d*) Sketch the curve with equation $y = \mathrm{f}(x)$ for $0 \leq x \leq \pi$.

(2)
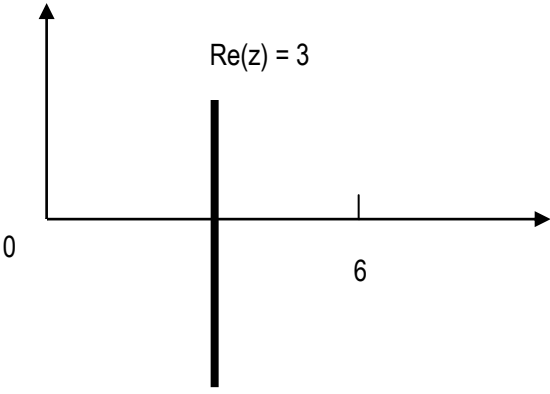
TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme	Marks
1(a)	$$\dfrac{1}{3r-1}-\dfrac{1}{3r+2}$$	M1 A1 (2)
(b)	$$\sum_{r=1}^{n}\dfrac{3}{(3r-1)(3r+2)}=\dfrac{1}{2}-\dfrac{1}{5}+\dfrac{1}{5}-\dfrac{1}{8}+\dfrac{1}{8}-\dfrac{1}{11}+\dots\dfrac{1}{3n-1}-\dfrac{1}{3n+2}$$ $$=\dfrac{1}{2}-\dfrac{1}{3n+2}=\dfrac{3n}{2(3n+2)}\quad *$$	M1 A1ft A1 (3)
(c)	$\text{Sum} = f(1000) - f(99)$ $$\dfrac{3000}{6004}-\dfrac{297}{598}=0.00301\quad\text{or }3.01\times10^{-3}$$	M1 A1 (2) **7**
2	$f''(t)=-x-\cos x,\qquad f''(0)=-1$ $f'''(t)=(-1+\sin x)\dfrac{\mathrm{d}x}{\mathrm{d}t},\qquad f'''(0)=-0.5$ $f(t)=f(0)+tf'(0)+\dfrac{t^2}{2}f''(0)+\dfrac{t^3}{3!}f'''(0)+\dots$ $\qquad =0.5t-0.5t^2-\tfrac{1}{12}t^3+\dots$	B1 M1A1 M1 A1 **5**

Question Number	Scheme	Marks
3(a)	$(x+4)(x+3)^2 - 2(x+3) = 0$, $(x+3)(x^2+7x+10) = 0$ so $(x+2)(x+3)(x+5) = 0$ or alternative method including calculator	M1
	Finds critical values –2 and -5	A1 A1
	Establishes $x > $ -2	A1ft
	Finds and uses critical value –3 to give $–5 < x < $ -3	M1A1 (6)
(b)	$x > $ -2	B1ft (1) **7**
4(a)	Modulus = 16	B1
	Argument = $\arctan(-\sqrt{3}) = \dfrac{2\pi}{3}$	M1A1 (3)
(b)	$z^3 = 16^3(\cos(\dfrac{2\pi}{3}) + i\sin(\dfrac{2\pi}{3}))^3 = 16^3(\cos 2\pi + i\sin 2\pi) = 4096$ or 16^3	M1 A1 (2)
(c)	$\overset{\frac{dy}{dx}}{}\quad w = 16^{\frac{1}{4}}(\cos(\dfrac{2\pi}{3}) + i\sin(\dfrac{2\pi}{3}))^{\frac{1}{4}} = 2(\cos\left(\dfrac{\pi}{6}\right) + i\sin\left(\dfrac{\pi}{6}\right))\ \left(= \sqrt{3} + i\right)$	M1 A1ft
	OR $-1 + \sqrt{3}i$ OR $-\sqrt{3} - i$ OR $1 - \sqrt{3}i$	M1A2(1,0) (5) **10**

Question Number	Scheme	Marks
5(a)	$1.5 + \sin 3\theta = 2 \quad \rightarrow \quad \sin 3\theta = 0.5 \quad \therefore 3\theta = \dfrac{\pi}{6} \left(\text{or} \ \dfrac{5\pi}{6} \right),$ and $\therefore \theta = \dfrac{\pi}{18}$ or $\dfrac{5\pi}{18}$	M1 A1, A1 (3)
(b)	$\text{Area} = \dfrac{1}{2}\left[\displaystyle\int_{\frac{\pi}{18}}^{\frac{5\pi}{18}} (1.5 + \sin 3\theta)^2 \, \mathrm{d}\theta \right], \ -\dfrac{1}{9}\pi \times 2^2$ $\overset{\frac{\mathrm{d}y}{\mathrm{d}x}}{=} \dfrac{1}{2}\left[\displaystyle\int_{\frac{\pi}{18}}^{\frac{5\pi}{18}} (2.25 + 3\sin 3\theta + \tfrac{1}{2}(1 - \cos 6\theta)) \, \mathrm{d}\theta \right] - \dfrac{1}{9}\pi \times 2^2$ $= \dfrac{1}{2}\left[\left(2.25\theta - \cos 3\theta + \tfrac{1}{2}(\theta - \dfrac{1}{6}\sin 6\theta)\right) \right]_{\frac{\pi}{18}}^{\frac{5\pi}{18}} - \dfrac{1}{9}\pi \times 2^2$ $= \dfrac{13\sqrt{3}}{24} - \dfrac{5\pi}{36}$	M1, M1 M1 M1 A1 M1 A1 (7) **10**

Question Number	Scheme	Marks
6(a)	Imaginary Axis Re(z) = 3 0 6 Real axis Vertical Straight line Through 3 on real axis	B1 B1 (2)
(b)	These are points where line $x = 3$ meets the circle centre (3, 4) with radius 5. The complex numbers are 3 + 9i and 3 – i.	M1 A1 A1 (3)
(c)	$\left\|z-6\right\| = \left\|z\right\| \Rightarrow \left\|\frac{30}{w}-6\right\| = \left\|\frac{30}{w}\right\|$ $\therefore \left\|30-6w\right\| = \left\|30\right\| \Rightarrow \therefore \left\|5-w\right\| = \left\|5\right\|$ This is a circle with Cartesian equation $(u-5)^2 + v^2 = 25$	M1 M1 A1 M1 A1 (5) **10**

4

Question Number	Scheme	Marks
7(a)	$\dfrac{dy}{dx} = \dfrac{dy}{dz} \cdot \dfrac{dz}{dx}$ and $\dfrac{dy}{dz} = 2z$ so $\dfrac{dy}{dx} = 2z \cdot \dfrac{dz}{dx}$ Substituting to get $2z \cdot \dfrac{dz}{dx} - 4z^2 \tan x = 2z$ and thus $\dfrac{dz}{dx} - 2z \tan x = 1$ $\ast$	M1 M1 A1 M1 A1 (5)
(b)	$\text{I.F.} = e^{\int -2\tan x\, dx} = e^{2\ln\cos x} = \cos^2 x$ $\therefore \dfrac{d}{dx}\left(z\cos^2 x\right) = \cos^2 x \quad \therefore z\cos^2 x = \int \cos^2 x\, dx$ $\therefore z\cos^2 x = \int \tfrac{1}{2}(\cos 2x + 1)\, dx = \tfrac{1}{4}\sin 2x + \tfrac{1}{2}x + c$ $\therefore z = \tfrac{1}{2}\tan x + \tfrac{1}{2}x\sec^2 x + c\sec^2 x$	M1 A1 M1 M1 A1 A1 (6)
(c)	$\therefore y = (\tfrac{1}{2}\tan x + \tfrac{1}{2}x\sec^2 x + c\sec^2 x)^2$	B1ft (1)
		12

Question Number	Scheme	Marks
8(a)	Differentiate twice and obtaining $$\frac{dy}{dx} = \lambda \sin 5x + 5\lambda x \cos 5x \text{ and } \frac{d^2y}{dx^2} = 10\lambda \cos 5x - 25\lambda x \sin 5x$$	M1 A1
	Substitute to give $\lambda = \dfrac{3}{10}$	M1 A1 (4)
(b)	Complementary function is $y = A\cos 5x + B\sin 5x$ or $Pe^{5ix} + Qe^{-5ix}$	M1 A1
	So general solution is $y = A\cos 5x + B\sin 5x + \dfrac{3}{10}x\sin 5x$ or in exponential form	A1ft (3)
(c)	$y = 0$ when $x = 0$ means $A = 0$	B1
	$\dfrac{dy}{dx} = 5B\cos 5x + \dfrac{3}{10}\sin 5x + \dfrac{3}{2}x\cos 5x$ and at $x = 0$ $\dfrac{dy}{dx} = 5$ and so $5 = 5A$	M1 M1
	So $B = 1$	A1
	So $y = \sin 5x + \dfrac{3}{10}x\sin 5x$	A1 (5)
(d)		"Sinusoidal" through O amplitude becoming larger B1 Crosses x axis at $\dfrac{\pi}{5}, \dfrac{2\pi}{5}, \dfrac{3\pi}{5}, \dfrac{4\pi}{5}$ B1 (2) **14**